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An experimental feasible scheme is proposed to generate Greenberger-Horne-Zeilinger (GHZ) type of maximal entanglement. This entanglement can be between either atomic ensembles or individual photons according to the difference applications we desire for it. The collective states of the atomic ensembles have the inherent robust to realistic noise and imperfections, which is the main requirements for the stationary qubit of quantum computation and information. While the individual photon is a suitable candidate for flying or transmitting qubit of the quantum computation and information as the photon polarization states can be manipulated conveniently with linear optics. Furthermore, the physical requirements of the scheme are moderate and well fit the experimental techinque. The preparation efficiency can be also 10^3 higher than the protocols such as multi-party entanglement preparation scheme based on spontaneous parametric down converter.

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Entanglement of many parties is of fundamental interest to test quantum mechanics against local hidden theory [1,2]. Furthermore, it has practical applications in various quantum information processing such as quantum crytography [3], computer [4], and teleportation [5]. And it is believed that with more subsystems entangled, quantum nonlocality becomes more striking [1,6], and the entanglement is more useful in actual applications [7–9]. Then great attention has been recently directed to get more and more subsystems entangled. In theory, many schemes have proposed to generate multi-party entanglement with cavity [10,11], ion traps [12] and spontaneous parametric down converter (SPDC) [13,14]. There are also proposals to entangle atomic ensembles [15] or indistinguishable atoms in Bose-Einstein condensates [16]. In experiment, there are reports of demonstration of four-photon entanglement in SPDC [14] and four-particle entanglement in ion traps [17].

In most of the experimental efforts, the subsystems in entangled system belong the same congener although experimentally feasible scheme has been proposed to entangle different congeners as atomic ensembles and photon [18]. Here, we propose a scheme to generate GHZ type of maximal entanglement with the following features: firstly, the entangled subsystems can be either atomic ensembles or individual photons, according to the difference application. Secondly, the physical requirements of this scheme are moderate and well fit the current experimental technique. Its efficiency can be also 10^3 higher than the multi-particle preparation protocols such as spontaneous parametric down converter.

The basic element of our scheme is an ensemble of many identical atoms, as in the papers [15,18], whose experimental realization can be either a room-temperature atomic gas or a sample of cold trapped atoms. The relevant structure of the atom is shown in Fig. 1. From the three levels $|g\rangle, |r\rangle, |l\rangle$, we define two collective atomic operations $s = (1/\sqrt{N^a}) \sum_{i=1}^{N^a} |g\rangle_i \langle s|$ with $s = r, l$, where $N^a \gg 1$ is the total atom number. The atoms are initially prepared through optical pumping to the ground state $|g\rangle$,which is effectively a vacuum state $|0\rangle$ of the operators r, l. A basis of the polarization qubits can be defined from the states $|R\rangle = r^\dagger |0\rangle$ and $|L\rangle = l^\dagger |0\rangle$, which have an experimentally demonstrated long coherence time [19–22]. In Raman processing, the atomic ensemble is transited from the ground state $|g\rangle$ to the state $|e\rangle$ by the classical laser (the pump light) with Rabi frequency Ω, and then is transited the two metastable states $|r\rangle$ and $|l\rangle$ with equal probability. From those transitions $|e\rangle \rightarrow |r\rangle$ and $|e\rangle \rightarrow |l\rangle$, two Stokes photons are emitted, which are horizontal and vertical polarization respectively. Due to the collectively enhanced coherent interaction, these excitation modes r and l can be transferred to optical excitation mode h and v respectively with high precision, and then detected by single-photon detectors, even for a free-space ensemble, which has been demonstrated in both in theory [23] and in experiments [20,21].

As pointed out in the paper [18], EPR entanglement state$\left|\Psi^1\right\rangle = (r_a^\dagger h_p^\dagger + l_a^\dagger v_p^\dagger)/\sqrt{2}\,|0\rangle_{ap}$ between atomic ensemble and Stokes photon can be prepared with a short off-resonant laser pulse in this system. This preparation has a controllable small probability p. In fact, the whole state of the atomic ensemble and the Stokes photons can be written in the form

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$$\left|\Psi\right\rangle_1 = \left|0\right\rangle_{ap} + \sqrt{p}\left|\Psi^1\right\rangle_1 + p\left|\Psi^2\right\rangle_1 + o(p^{3/2}) \tag{1}$$

where$\left|0\right\rangle_{ap}$ is the vacuum state of the whole system, $\left|\Psi^1\right\rangle$ and $\left|\Psi^2\right\rangle = ((r_{a_1}^\dagger h_{p_1}^\dagger)^2 + (l_{a_2}^\dagger v_{p_1}^\dagger)^2 + r_{a_1}^\dagger h_{p_1}^\dagger l_{a_2}^\dagger v_{p_1}^\dagger)/\sqrt{3}\left|0\right\rangle_{ap}$ are the states involving one and two Stokes photon respectively. Here $h^\dagger(v^\dagger)$ 7represents horizontal (vertical) mode creation operator of Stokes photon. $o(p^{3/2})$ denotes the terms with more than two excitations whose probabilities are smaller than p and we can reasonably neglect it in the following discuss. We have also neglected the fixed phase difference in the states $\left|\Psi^1\right\rangle$ and $\left|\Psi^2\right\rangle$. As shown in the paper, the preparation of this state, with inherent resilience to noise, is well based on the current technology of laser manipulation. Many application such as quantum memory can be expected from this novel entanglement between atomic collective state and the individual photon polarization state [18].

In order to generate multi-party entanglement, we can at first prepare n pair entanglement between atomic ensembles and Stokes photons. A simply way is to illuminate n atomic ensembles in turn with a pump classical light. Then the whole system of the atomic ensembles and Stokes photons is in the state

$$\left|\Phi\right\rangle^{ap} = \prod_{i=1}^{n}\left|\Psi\right\rangle_i = \prod_{i=1}^{n}(\left|0\right\rangle_{ap} + \sqrt{p}\left|\Psi^1\right\rangle_i + p\left|\Psi^2\right\rangle_i) \tag{2}$$

In the expansion of this state, there are $T = \sum_{m=0}^{\lfloor\frac{n}{2}\rfloor}\frac{n!}{m!m!(n-2m)!}3^m \times 2^{n-2m}$ components involving n Stokes photons with the same probabilities amplitude $p^{n/2}$, where $\lfloor\frac{n}{2}\rfloor$ represents the integer part of $\frac{n}{2}$, and $0! = 1$. From this state combining n atomic ensembles and n Stokes photon, we can then prepare multi-particle entanglement.

For the generation of multi-atomic-ensemble entanglement, we can measure the n Stokes photons of the state $\left|\Phi\right\rangle^{ap}$ in the basis $\left|M\right\rangle^{\pm} = (1/\sqrt{2})\prod_{i=1}^{n}(\left|h\right\rangle_i^p \pm \left|v\right\rangle_i^p)\left|0\right\rangle^p$ which can be realized with the optical setup as Fig. 3. In this n-photon GHZ analyzer, if there is a photon in each output before the $\lambda/2$ plates and there are coincidence counts between even or odd D_V^i detectors, then the n photons are measured in the state $\left|M\right\rangle^+$ or $\left|M\right\rangle^-$. Note that for any practical application of the multi-party entanglement, the state preparation should be succeeded by a measurement of the polarization of the excitation on each ensemble [24,6–8]. It means we only post-select the cases that excitation appears on each ensemble (then there is Stokes photon in each input of the above measurement optical setup). In the case that n Stokes photon of the state $\left|\Phi\right\rangle^{ap}$ is measured in the basis $\left|M\right\rangle^{\pm}$, the residual n atomic ensembles are collapse into the state

$$\begin{aligned}\left|\Psi\right\rangle^a &= \left\langle M^{\pm}\right|\left.\Phi\right\rangle^{ap} \\ &= (1/\sqrt{2})(\prod_{i=1}^{n}\left|r\right\rangle_i^a + \prod_{i=1}^{n}\left|l\right\rangle_i^a)\left|0\right\rangle^a\end{aligned} \tag{3}$$

This is n-atomic-ensemble GHZ type maximal entanglement we can post-select for any application. It should be pointed out that we have neglect the cases where there are also n photons, but some atomic ensembles are not excited. With the inherent tolerance to realistic noise, the reserve of this n-atomic-ensemble GHZ entanglement can be much easier and longer. Many application as quantum computation employing multi-party entanglement and Bell measurement [8] can be expected for this entanglement state of stationary or storing qubits.

As we can transfer the excitation modes r and l of the n atomic ensembles to optical excitation modes h and v, the entanglement $\left|\Psi\right\rangle^a$ between n atomic ensembles can be transferred to n photons. Otherwise, we can measure those n optical excitation transferred from the n atomic ensembles of the state $\left|\Phi\right\rangle^{ap}$with the same setup as above in Fig. 3 to straightly project the n Stoke photons in corresponding GHZ state. In the case that those optical modes are measured in the state $\left|M\right\rangle^{\pm}$ which equals to measure the n atomic ensembles in the basis $\left|N\right\rangle^{\pm} = (\prod_{i=1}^{n}\left|r\right\rangle_i^a \pm \prod_{i=1}^{n}\left|l\right\rangle_i^a)/\sqrt{2}\left|0\right\rangle^a$, the remaining n Stokes photons of the state $\left|\Phi\right\rangle^{ap}$ are projected into state

$$\begin{aligned}\left|\Psi\right\rangle^p &= \left\langle N^{\pm}\right|\left.\Phi\right\rangle^{ap} \\ &= (1/\sqrt{2})\prod_{i=1}^{n}(\left|h\right\rangle_i^p \pm \left|v\right\rangle_i^p)\left|0\right\rangle^p\end{aligned} \tag{4}$$

This is exactly n-photon GHZ type maximal entanglement.

Then with the present scheme, we can prepare either n-atomic-ensemble or n-photon entanglement. On the one hand, the inherent resilience to noise make of the collective states of atomic ensemble makes it a well qualified candidate for stationary and register qubits of quantum information and computation. On the other hand, the light is an ideal carrier of quantum information and the individual mode as the polarization of photon has the advantage of convenient manipulation. According to the different applications and our own, we can choose the entangled subsystems as either atomic ensembles or individual photons. Although it is not exact multi-party GHZ state, it can yield effectively GHZ entanglement with post-selection, and then various applications such as quantum computation employing multi-party entanglement [8] and multi-man quantum cryptograhy.

We now give a brief discussion on the efficiency and the practical implementation of this proposal. As shown in the paper [15], control the probability of getting a Stokes photon $p \sim 10^{-2}$for a Raman driving pulse with short a light-atom interaction time t_{Δ}. With the collapsing measurement $|M\rangle$ or $|N\rangle$, the probability of eventually getting the n-party GHZ entangled state of individual photons or atomic ensembles is $q \sim 2p^n/T \sim 10^{-2n}/\sum_{m=0}^{\lfloor \frac{n}{2} \rfloor} \frac{n!}{m!m!(n-2m)!} 3^m \times 2^{n-2m-1}$. For example, the efficiency of generating three-photon GHZ entanglement is of the order of 10^{-7} which has improved 10^3 compared with that of post-selection preparing procedure in SPDC [13]. Then with a typical repetition frequency $f_p = 10^7 Hz$, we can get a free multi-party GHZ entanglement per second. In the practical implementation, we can safely neglect the dark counts of the single-photon detectors in the coincidence detections [13]. The transfer of the atomic ensemble excitation mode to the optical mode can be high efficient. As long as all the preparing operations are accomplished in a time no longer that the coherence times of the atomic collective states $T_{pre} \preceq ms$, we can also safely neglect the noise of the non-stationary phase drift induced by the pumping lase or by the optical channel.

In conclusion, we have proposed an experimentally feasible scheme to prepare multi-party GHZ type of maximal states between either atomic ensembles or individual photons according to the difference applications. The physical requirements of this scheme are moderate and well fit for the current experimental technique, its preparation efficiency can be also 10^3 higher than the SPDC multi-particle generation protocol.

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Figure Captions:

Figure 1: The relevant level structure with $|g\rangle$, the ground state, $|e\rangle$, the excited state, and $|r\rangle$, $|l\rangle$ the two metastable state for storing a qubit. The transition $|g\rangle \rightarrow |e\rangle$ is coupled by a classical laser (the pump light) with Rabi frequency Ω, followed with the equal probabilities transitions $|e\rangle \rightarrow |r\rangle$ and $|e\rangle \rightarrow |l\rangle$ where right-handed and left-handed rotation forward-scattered Stokes photons are emitted respectively. For convenience, we assume off-resonant coupling with a large detuning Δ.

Figure 2: Schematic drawing of the experimental setup for the generation of the entanglement state$|\Phi\rangle^{ap}$. The frequency-selective filter separate the pump light from the Stokes photon.

Figure 3: Schematic of the experimental setup for the measurement $|M\rangle^{\pm} = (1/\sqrt{2}) \prod_{i=1}^{n} (|h\rangle_i^p \pm |v\rangle_i^p) |0\rangle^p$of the n Stokes photon in the state$|\Phi\rangle^{ap}$. The polarizing beam splitters (PBS) reflect vertical photons and transmit horizontal photons. We can adjust the arrival time of the n photons with the delay plates. The $\lambda/2$ plates are employed to rotate the polarization of the Stokes photon i through 45^0 to transfer the original H/V basis as $|H'\rangle = (|H\rangle + |V\rangle)/\sqrt{2}$ and $|V'\rangle = (|H\rangle - |V\rangle)/\sqrt{2}$, where λ is the wavelengh of those photons. In necessary, we can also place narrow bandwidth filters before the single-photon detectors to earse the time difference between those photons.

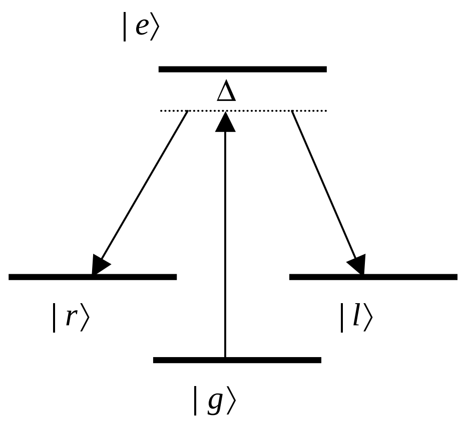

Fig. 1. Guo

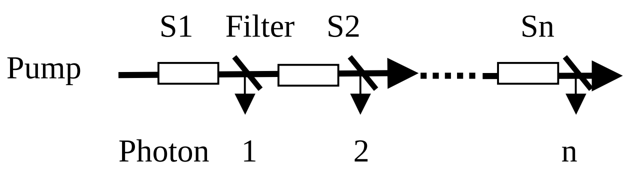

Fig. 2. Guo

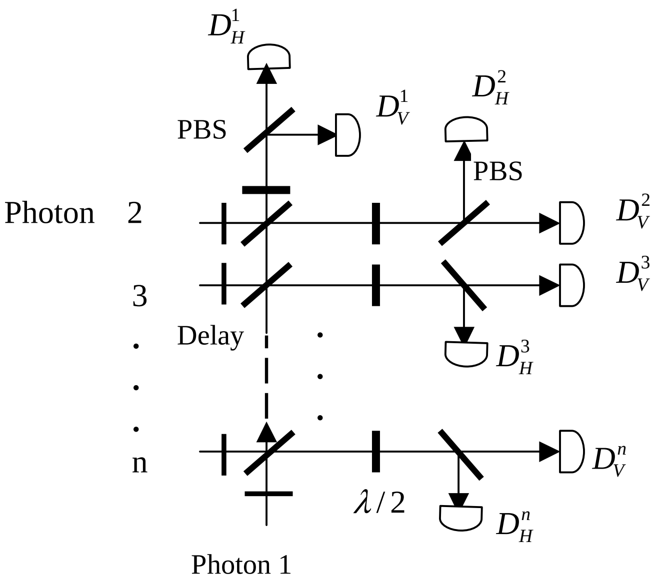

Fig. 3. Guo